Two Formal Representations of the Thematic-Rhematic Structure of Sentences

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Abstract

In this paper, we give two formal representations of the thematic-rhematic (T-R) structure of a natural language discourse. One is based on the concept of ontological promiscuity and the other based on a typed λ-calculus.

1 Introduction

In this paper, we give two formal representations of the thematic-rhematic (T-R) structure of a natural language discourse. Some pairs, triples, or in generally n−tuples of sentences in a discourse may differ in the place of their information focus. The distribution of this information focus is called the thematic-rhematic (T-R) structure or dichotomy. Besides "theme and rheme", similar terms as "old-information and new-information", "topic and comment", *etc.* are used in the literature concerning functional linguistics. We treat these concepts formally using two different tools. The tool used in Section 2 is a logical notation which is first-order and nonintensional. Using this description, a proof process of utterance interpretation of a discourse can be obtained. Implicit definition of the concepts of *theme* and *rheme* is given axiomatically. In the conventional logical notation, it is difficult to represent the information concerning the T-R dichotomy of sentences while such an information is important for the analysis of a discourse. It is shown that this is related to the problem of ontology. If, however, we abandon ontological scruples, it become easier to represent the T-R structure. We use the concept of ontological promiscuity proposed by J. R. Hobbs. In Section 2, using this notion, it is shown that we can give an account for utterance interpretation as a proof process. It is also shown that the *closed world assumption* (CWA) is essentially related to the proposed proof prcess. In Section 3, we consider the same problem mainly for Japanese, using another tool. We propose to use typed λ-calculus to analyse the problem. A logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand for a theme part and a rheme part of a sentence, respectively. The difference of T-R dichotomy is given by different types. Thus the same sentence may have different types depending on the situation. For utterances, type inference will be performed. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. In Section 3, we elaborate on this idea. In Section 4, conclusions are given. Sections 2 and 3 can be read independently except the discussins on the pragmatic problem. The numbering of formulae is *local* in each section.

2 Representation based on ontological promiscuity

In this section, a formal description of the thematic-rhematic (T-R) dichotomy of sentences is given based on a logical notation which is first-order and nonintensional. Using this description, a proof process of discourse understanding can be obtained. Besides "theme and rheme", similar terms as "old-information and new-information", "topic and comment", *etc.* are used in the literature concerning functional linguistics. However, since we do not define these terms explicitly, it is not essential which terms are used. In our analysis, we give implicit definition of these concepts axiomatically. The logical notation used in our analysis is

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the concept of ontological promiscuity proposed by J. R. Hobbs. In the conventional logical notation, in particular, in the first order predicate logic, it is difficult to represent the information concerning the T-R dichotomy of sentences. We can show that this is related to the problem of ontology. Let us consider the following sentence.

$$John\ is\ a\ student. \tag{1}$$

The meaning of (1) may be represented by the following predicate logic:

$$student(John). \tag{2}$$

Meanwhile the above sentence (1) may have three possible T-R dichotomies. If sentence (1) is uttered as an answer to the question "What is John?", "John" is known and considered as a theme, and "is a student" will be a rheme part. Thus, the T-R dichotomy of (1) is given as follows:

$$\frac{John}{T}\ \frac{is\ a\ student.}{R} \tag{3}$$

$$(John\ wa\ gakusei\ des-u.)$$

where T and R stand for *Theme* and *Rheme*, respectively. We annotate a Japanese translation for comparison if there exists an adaquate corresponding Japanese sentence. Note, in particular, the uses of postpositions *wa* and *ga*. On the other hand, if sentence (1) is uttered to the question "Who is a student?", "John" will be a rheme part and the T-R dichotomy is given by

$$\frac{John}{R}\ \frac{is\ a\ student.}{T} \tag{4}$$

$$(John\ ga\ gakusei\ des-u.)$$

(4) is equivalent to "It is John who is a student." There is another possible T-R dichotomy given by

$$\frac{John\ is\ a\ student.}{R} \tag{5}$$

In the above case, whole the sentence will be a rheme, i.e., a new information. In the discourse analysis, the information concerning the T-R dichotomy as described above is important. However, the conventional logical form as (2) cannot represent the information as (3)-(5). If we write $student(John)$ for (3), (4) may be writen as $John(student)$. However, this is not allowed, at least for the first order predicate logic, since "John" and "student" are of different type (ontology) and can not be used as an individual and a predicate simultaniously. However, if we abandon ontological scruples, it becomes easier to represent the T-R structure in the logical form. We propose to use the concept of ontological promiscuity proposed by J. R. Hobbs. In his notation, a prime operator (or nominalization operator) is introduced. For example, for $student(John)$, the following first-order predicate is defined:

$$student'(E, John). \tag{6}$$

(6) can be interpreted as "E is an event (condition, utterence) that John is a student". In this notion "John" and "E" become arguments of the same predicate $student'$ while they belong to different categories. For this reason this notion is called the ontological promiscuity by Hobbs. Note that (6) states only that "E is an event that John is a student" and is not equivalent to (2). For (6) to imply (2), the event E must exist in the real world. Let $exist(E)$ denote that E exists in the real world. Then (2) will be implied by

$$student'(E, John) \wedge exist(E). \tag{7}$$

(7) can be interpreted as "E is an event that John is a student and E exists in the real world". In (7), E is not needed to be specific and thus (2) will be equivalent to

$$(\exists e)\ student'(e, John) \wedge exist(e). \tag{8}$$

Thus, in general, the prime operator is defined for a predicate with n arguments by the following axiom scheme.

$$(\forall x_1, \cdots, x_n)\ p(x_1, \cdots, x_n)$$
$$\equiv (\exists e)\ p'(e, x_1, \cdots, x_n) \wedge exist(e) \tag{9}$$

Using this notion, a predicate is nominalized and can be refered to in another predicate. We now describe the T-R dichotomy by using the prime operator. We introduce four predicates $theme(\cdot,\cdot)$, $rheme(\cdot,\cdot)$, $p\_theme(\cdot)$, and $p\_rheme(\cdot)$. For example, we express (3) as follows.

$$(\exists e)\ student'(e, John) \wedge exist(e)$$

$$\wedge theme(e, John) \wedge p\_rheme(e) \qquad (10)$$

The first two predicates represent that John is a student while $theme(e, John)$ says that in the event (or utterence) e "John" is a theme and $p\_rheme(e)$ says that in such an e, the predicate part "*is a student*" is a rheme. Similarly, the T-R dichotomy (4) can be represented as

$$(\exists e)\ student'(e, John) \wedge exist(e)$$

$$\wedge rheme(e, John) \wedge p\_theme(e) \qquad (11)$$

In (11), $rheme(e, John)$ says that in the event (or utterence) e "John" is a rheme and $p\_theme(e)$ says that in such an e, the predicate part "*is a student*" is a theme. The T-R dichotomy (5) can be represented by using p_rheme as follows.

$$(\exists e)\ student'(e, John) \wedge exist(e)$$

$$\wedge rheme(e, John) \wedge p\_rheme(e) \qquad (12)$$

In (12), $p\_rheme(e)$ means that in the utterence e, the predicate part (i.e., *is a student*) is a rheme. Generally, the T-R dichotomy of a predicate $p(x_1, \cdots, x_n)$ is represented as follows.

$$(\exists e)\ p'(e, x_1, \cdots, x_n) \wedge exist(e)$$

$$\wedge X_1(e, x_1) \wedge\ \cdots X_n(e, x_n) \wedge Y(e) \qquad (13)$$

where $X_1, \cdots, X_n$ take either *theme* or *rheme*, and Y takes either *p_theme* or *p_rheme*. Note that we can use another representation to express that the whole sentence is a rheme. For example, (5) can be expressed as

$$(\exists e)\ student'(e, John) \wedge exist(e)$$

$$\wedge rheme(e, e) \qquad (14)$$

$rheme(e, e)$ can be interpreted as "In the event e, e itself is a rheme, i.e., the whole sentence is a rheme". Thus (12) and (14) are considered to be equivalent. Generally, this observation is stated in the following axiom.

$$(\forall e, x_1, \cdots, x_n) p'(e, x_1, \cdots, x_n)$$

$$\wedge rheme(e, x_1) \wedge \cdots$$

$$\cdots \wedge rheme(e, x_n) \wedge p\_rheme(e) \equiv$$

$$p'(e, x_1, \cdots, x_n) \wedge rheme(e, e) \qquad (15)$$

In each sentence, there exists at least one rheme part. This axiom can be expressed as follows.

$$(\forall e, x_1, \cdots, x_n)[p'(e, x_1, \cdots, x_n) \wedge exist(e)] \supset$$

$$rheme(e, x_1) \vee \cdots \vee rheme(e, x_n)$$

$$\vee p\_rheme(e) \qquad (16)$$

We now apply the above formulation to the analysis of discourse (or text). Here we describe the discourse understanding model as a proof process. Our model works as an on-line reasoning procedure. In this model, we have axioms ((9),(15),(16),*etc.*) as described above. Each current utterance of the discourse is also considered to be an axiom. However these axioms must satisfy several constraints which we call the discourse rules. We set up these rules in the form of $\vdash A \supset B$ (see (18), (20), (26) below). This means that if A is a therem then B is also a theorem. If A can be deduced from the current utterance in the discourse (the newest axiom) plus the existing axioms, then B must hold. Here the existing axioms mean the axioms of the original system ((9),(15),(16),*etc.*) plus the utterances up to the preceding utterance which have been included into the system. If B is proved to hold, then the discourse up to the current utterance is correct, and the current utterance will be included into the existing set of axioms. If B can not be proved, then the discourse is considered to be incorrect. Our system checks wheather each constraint holds based on the existing axioms and the current utterance. Thus note that it is not necessary to number sentences appearing in the discourse. We consider a discourse or text in which themes and rhemes appear succesively as follows.

$$\begin{array}{llllll} 1st\ sentence: & T & - & R & & \\ & & & \vdots & & \\ 2nd\ sentence: & & & T & - & R \\ \vdots & & & & & \vdots \end{array}$$

This structure is observed typically in a story. For example, the following sentence can not be at the beginning of the text.

$$^{*}There\ is\ the\ bycicle. \qquad (17)$$

In English, the use of particles *the* and *a (an)* is deeply related to the T-R structure. In Japanese, the T-R dichotomy is well represented by postpositions *wa* and *ga*. Korean has a similar system. Meanwhile, in slavic languages as Czech, Polish and Russian, the word order is free and this degree of freedom is used for the representation of T-R dichotomy. In Chinese, the word order is also used for T-R dichotomy. In (17), "bycicle" is a theme and thus it should have appeared as a rheme preceding this sentence. However (17) is the first sentence in the discourse so this is impossible. This is generalized to the following rule.

$$\vdash\ (\forall e, x)\ theme(e, x)$$
$$\supset\ [(\exists e_1) \neg e_1 = \dot{e} \wedge rheme(e_1, x)] \qquad (18)$$

This says that if x is a theme of the event e then there exists another event e_1 in which x is a rheme. Note that, generally, a term x can not be a theme and a rheme of the same event e at the same time. We can write this rule as the following axiom.

$$(\forall e, x)\ theme(e, x)\ \supset\ \neg rheme(e, x) \qquad (19)$$

Then rule (18) can be simplified as follows.

$$\vdash\ (\forall e, x)\ theme(e, x)\ \supset\ [(\exists e_1) rheme(e_1, x)] \qquad (20)$$

Remark: For the predicates $p\_theme(\cdot)$ and $p\_rheme(\cdot)$, the following axiom and theorem hold. However we dot not treat these for simplicity.

$$(\forall e)\ p\_theme(e)\ \supset\ \neg p\_rheme(e)$$

$$\vdash (\forall e, x_1, \cdots, x_n)[p\_theme(e) \wedge p'(e, x_1, \cdots, x_n)] \supset$$
$$[(\exists e_1, y_1, \cdots, y_n)\ p\_rheme(e_1) \wedge p'(e_1, y_1, \cdots, y_n)]$$

Before acceptance of the first sentence of a discourse, the proof system consists of the axioms (9),(15),(16),*etc.* and theorem (20). Theorem (20) indeed holds since $theme(e, x)$ does not hold for any e, x if we adopt the *closed world assumption* (CWA). Let us consider the following pair of sentences.

There is a bycicle. (21.1)

(*Jitensha ga arimas − u.*)

The bycicle is new. (21.2)

(*Jitensha wa atarashii des − u.*)

In general, a noun with particle "*a*" constitutes a rheme part of the sentence that appears at the beginning of a text, while that noun with particle "*the*" appears in the second, third, *etc.* sentences as themes. Thus the logical notations for (21) can be adequately represented as follows:

$$(\exists e, x) bycicle'(e, x) \wedge exist(e) \wedge rheme(e, x) \quad (22.1)$$

$$(\exists e, \exists! x) bycicle(x) \wedge new'(e, x) \wedge exist(e) \wedge$$
$$theme(e, x) \qquad (22.2)$$

Here $\exists! x$ means that there exists only one x. The above discourse is correct and this is proved as follows. The logical expression (22.1) is included into the system as an new axiom. Then the system tries to prove theorem (20). By the CWA, the premise of (20) "$theme(e, x)$" does not hold for any e, x. Thus the system has suceeded in proving theorem (20) and the discourse is correct up to (22.1). The system adopts (22.1) as an axiom. Next the logical expression (22.2) is added to the system as a new axiom. Let e_0 and x_0 be constants that satisfy (22.2). Since only x_0 satisfies $bycicle(x)$, this x_0 also satifies (22.1). From (22.1), the premise of theorem (20) holds. However, by (22.1) and (19), we have $rheme(e_1, x_0)$ for some e_1. Therefore it can be concluded that the discourse (21) is correct. Sentence (17) at the beginning of the text will be rejected in the following way. First note that since "the" is attached to "bycicle", the logical form of (17) can be given by the following form.

$$(\exists e, x) bycicle'(e, x) \wedge exist(e) \wedge theme(e, x) \quad (23)$$

When sentence (17) is accepted in the system, the logical expression (23) corresponding to (17) is added to the system. Next the system tries to check theorem (20). The premise of theorem (20) now holds by (23), however the conclusion "$(\exists e_1) rheme(e_1, x)$" of theorem (20) never holds by the CWA. Thus sentence (17) has been rejected and the discourse is not correct. Our formulation can be used in the analysis of a pragmatic problem. To see this we now consider the following example.

There is a bycicle. (24.1)

(*Jitensha ga arimas − u.*)

The saddle is new. (24.2)

(*Sadoru wa atarashii des − u.*)

This pair of sentences is natural although "the saddle" appearing as a theme does not satisfy rule (20). This is because a saddle is a part of a bycicle. However the following pair seems to be a little strange.

$$\textit{There is a bycicle.} \tag{25.1}$$

$$(\textit{Jitensha ga arimas} - u.)$$

$${}^{?}\textit{The radio is new.} \tag{25.2}$$

$$({}^{?}\textit{Rajio wa atarashii des} - u.)$$

In (25.2) "the radio" is a theme. However a usual bycicle does not posecess a radio. Thus it does not seem to be natural that "the radio" appears in the discourse for the first time as a theme. If there is a sentence as "*The bycicle has a radio* (*a radio* : *rheme*)" between (25.1) and (25.2), then the whole discourse may be more acceptable. In general, it is observed that if x is a part of y that has already appeared as a rheme, then x can be a theme. This observation can be summarised as the following rule, which is a modification of rule (20).

$$\vdash\ (\forall e, x)\ theme(e,x)\ \supset$$

$$\Big([(\exists e_1) rheme(e_1, x)] \vee$$

$$[(\exists y, e_2) part\_of(x,y) \wedge rheme(e_2, y)]\Big) \tag{26}$$

Here $part\_of(x,y)$ means that x is a part of y. We add the following axioms.

$$(\forall x,y)[saddle(x) \wedge bycicle(y)] \supset part\_of(x,y) \tag{27.1}$$

$$(\forall x,y)[radio(x) \wedge bycicle(y)] \supset \neg part\_of(x,y) \tag{27.2}$$

The logical form for (24.1) is the same as (22.1) and (24.2) is translated into the following form.

$$(\exists e_1, \exists! y) saddle(y) \wedge new'(e_1, y)$$

$$\wedge exist(e_1) \wedge theme(e_1, y) \tag{28}$$

Theorem (26) and the correctness of the discourse can be proved from axioms (22.1), (27.1) and (28). Now let us consider (25). The logical form of (25.1) is the same as (22.1) and the logical form for (25.2) is given as follows.

$$(\exists e_1, \exists! y) radio(y) \wedge new'(e_1, y)$$

$$\wedge exist(e_1) \wedge theme(e_1, y) \tag{29}$$

However, from (22.1) and (29), theorem (26) cannot be proved. This establishes that discourse (25) is not acceptable.

3 Representation Based on a Typed λ-Calculus

The purpose of this section is to propose a formal model for utterance interpretation of the thematic-rhematic structure of a Japanese sentence using a typed λ-calculus. In our analysis, a logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand for a theme part and a rheme part of a sentence, respectively. Although we analyse mainly Japanese sentences, the results can be applied to other languages. The T-R dichotomy of a Japanese sentence is represented by the postpositions *wa* and *ga*. For example, the following two sentences are different in T-R dichotomy, and used in diffrent situations: (*a*) *Taroo wa Gakusei des-u.* (*Speaking of Taroo, he is a student.*) (*b*) *Taroo ga Gakusei des-u.* ((*Of all the people we are talking about*) *Taroo* (*and only Taroo*) *is a student.*) The meaning of both (a) and (b) is "Taroo is a student", and thus may be writen as $student(Taroo)$. However this representation is obviously not sufficient for an account of the utterance interpretation of (a) and (b). The NP (noun phrase) of (a) marked with *wa* functions as a theme, i.e., it should have already appeared in the preceding discourse and thus can be considered as an old information. Therefore, in the discourse, sentence (a) should be preceded by a sentence that contains *Taroo* as a rheme (new information). For example, *Taroo* in the following sentence can be considered as a new information: (c) *Taroo ga ima-su.* (*Here is Taroo.*) The pair (c), (a) in this order is a correct discourse utterance. On the other hand, the pair (c), (b) cannot be considered as correct since *student* functions as a theme in (b) while it has not appeared in the preceding context. As is seen from (b) and (c), an NP marked with postposition *ga* functions as a rheme (i.e., information focus). To explain the difference between (a) and (b) in the utterance level, we annotate $\lambda x.student(x)$ of (a) and (b) by different typed λ-terms. Roughly speaking we assign $T \rightarrow R$ and R to each $\lambda x.student(x)$ of (a) and (b), respectively.

Baesd on this, if we can show $student(Taroo) : R$ then we say sentence (a) (or (b)) of the discourse is correct. For example, if $Taroo$ of (a) has a type T then by the β-reduction of typed λ-calculus, we have $student(Taroo) : R$. For $Taroo$ to have a type T, we impose a constraint that $Taroo$ must have appeared in a preceeding sentence. Other cases can be treated similarly. See the following descriptions for details. Thus the correctness of the discourse can be proved by checking the correctness of the types of each formula. In general, given a discourse $s_0, s_1, \cdots, s_n$ in logical forms, what we have to show is that $(\vdash s_0 : R)$, $(s_0 : R \vdash s_1 : R)$, $\cdots$, $(s_0 : R, \cdots, s_{n-1} : R \vdash s_n : R)$, succesively. In this section we elaborate on this idea.

First consider the following discourse consisted of a single sentence.

$$Taroo\ ga\ imas-u.\ (here\ is\ Taroo.) \tag{1}$$

The meaning of this sentence is:

$$s_0 = here\_is(Taroo) \tag{2}$$

We define this discourse to be *correct* if $s_0 : R$. This is done in the following way: Translate "*Taroo ga*" into $\lambda f.f(Taroo)$. We let this formula have either type of $T \rightarrow R$ or $R \rightarrow R$ when the proper noun "Taroo" is marked with the postposition "ga". Thus we have the following translation rules:

$$\text{"}Taroo\ ga\text{"} \Longrightarrow \lambda f.f(Taroo) \in s_0 : T \rightarrow R \tag{3.1}$$

$$\text{"}Taroo\ ga\text{"} \Longrightarrow \lambda f.f(Taroo) \in s_0 : R \rightarrow R \tag{3.2}$$

This can be writen for short as

$$\text{"}Taroo\ ga\text{"} \Longrightarrow \lambda f.f(Taroo) \in s_0 : (T, R) \rightarrow R \tag{4}$$

In the above, $t \in s_0$ means that t is a typed λ-term component of the logical formula s_0. That is

$$t \in s_0 \ \ iff \ \ (\exists t_1, t_2)\ t_1 t t_2 = s_0 \tag{5}$$

Here t_1 and/or t_2 may be empty. Thus $s_0 \in s_0$. From (3), we have

$$\vdash \lambda f.f(Taroo) \in s_0 : (T, R) \rightarrow R \tag{6}$$

The verb "imas-u" allows a neutral description. A neutral description has the following T-R dichotomy:

$$\frac{Taroo\ ga}{Rheme}\ \frac{imas-u.}{Rheme} \tag{7}$$

A sentence of neutral description in the Japanese language was first found and named by Kuroda. This kind of sentence has no theme part. For this kind of verb, we assign a type R and write as follows:

$$\vdash \lambda x.here\_is(x) \in s_0 : R \tag{8}$$

Now by (6) and (8) we can deduce the following *judgement.*

$$\begin{aligned} &e_0 : A_0, e_1 : A_1 \vdash \\ &(\lambda f.f(Taroo))(\lambda x.here\_is(x)) \\ &(\lambda x.here\_is(x))(Taroo) \\ &= here\_is(Taroo) = s_0 : R \end{aligned} \tag{9}$$

where $e_0 : A_0$ and $e_1 : A_1$ stand for (6) and (8), respectively. Thus $s_0 : R$ has been proved and the correctness of the discourse (1) has been established. To deduce (9), we have of course used the inference rule of the typed λ-calculus given by

$$e_0 : \alpha \rightarrow \beta, e_1 : \alpha \vdash e_0 e_1 : \beta \tag{10}$$

Note that the type used for $(\lambda f.f(Taroo))$ in deduction (9) is $R \rightarrow R$. In general, for a nuetral description, β-reduction for $R \rightarrow R$ and R occur. Next we consider the discourse consisted of the following two sentences.

$$Taroo\ ga\ imas-u.\ (here\ is\ Taroo.) \tag{11.1}$$

$$Taroo\ wa\ gakusei\ des-u.(Taroo\ is\ a\ student.) \tag{11.2}$$

The T-R dichotomies of the above sentences are as follows:

$$\frac{Taroo\ ga}{Rheme}\ \frac{imas-u.}{Rheme} \tag{12.1}$$

$$\frac{Taroo\ wa}{Theme}\ \frac{gakusei\ des-u.}{Rheme} \tag{12.2}$$

The NP (noun phrase) of (12.2) marked with *wa* functions as a theme. It should have already appeared in the preceding discourse as a rheme. The discourse (12) satisfies this constraint since "*Taroo*" appears as a rheme in (12.1) since it is marked with the postposition "*ga*". The discourse (12) is actually correct. We now formally state the correctness of (12). The logical forms of (12.1) and (12.2) are given as

$$s_0 = here\_is(Taroo) \tag{13.1}$$

$$s_1 = student(Taroo) \qquad (13.2)$$

First we must show $s_0 : R$, however we have already seen this. Thus we show $s_1 : R$. Note that $s_0 = (\lambda x.student(x))(Taroo)$. It is natural to assign $\lambda x.student(x)$ a type $T \rightarrow R$ since (12.2) contains the postposition "*wa*". This postposition is called the thematic "*wa*". We write this as follows.

$$\text{"}wa\ gakusei\ des - u\text{"} \Longrightarrow$$

$$\lambda x.student(x) \in s_1 : T \rightarrow R \qquad (14)$$

Thus we have

$$\vdash \lambda x.student(x) \in s_1 : T \rightarrow R \qquad (15)$$

Therefore if "*Taroo*" has a type T, we have $s_1 : R$. The NP can be a theme if it has already appeared in the preceding discourse as a rheme. This can be writen as follows:

$$\lambda f.f(Taroo) \in s_0 : (T, R) \rightarrow R \vdash Taroo \in s_1 : T \qquad (16)$$

Now $s_1 : R$ can be shown as follows. By (6) and (16),

$$\vdash Taroo \in s_1 : T \qquad (17)$$

Applying the β-reduction rule to (15) and (17), we have $s_1 : R$. Thus the discourse (11) is correct. In Japanese, the following sentence at the beginning of the discourse is not natural.

$$Taroo\ wa\ gakusei\ des - u.(Taroo\ is\ a\ student.) \qquad (18)$$

This is because "*Taroo*" appears as a theme but it is not preceeded by a sentence in which *Taroo* appears as a rheme. In our formal description, the incorrectness of the discourse (18) is described as a failure of type checking. We define the discourse to be incorrect if either $s_0 : R$ or $s_1 : R$ is not proved. Indeed, $s_0 : R$ where $s_0 = student(Taroo)$ is not proved since we do not have $Taroo \in s_0 : T$.

We now consider the following discourse consisted of two sentences.

$$Gakusei\ ga\ i - mas - u. \qquad (19.1)$$

$$Taroo\ ga\ gakusei\ des - u. \qquad (19.2)$$

The logical forms for (19.1) and (19.2) are given as follows.

$$s_0 = (\exists x)student(x) \wedge here\_is(x) \qquad (20.1)$$

$$s_1 = student(Taroo) \qquad (20.2)$$

Since "*gakusei* (*student*)" is marked with the postposition *ga*, and the verb $imas - u$ allows a neutral description, we have

$$(\exists x)student(x) \wedge here\_is(x) \in s_0 : R \qquad (21)$$

From this we have

$$(\exists x)student(x) \in s_0 : R \qquad (22)$$

In general we impose the following postulate.

$$A \wedge B \in s_i : R \vdash A \in s_i : R \qquad (23)$$

The logical connective $\wedge$ can be replaced by $\vee$. Furthermore we add the following postulate.

$$Qxf(x) \in s_0 : R \vdash \lambda x.f(x) \in s_1 : T \qquad (24)$$

where Q stands for a quantifier $\forall$ or $\exists$. This postulate means that a predicate that appeared as a rheme can be treated as a theme in the succeeding sentences. From this and (22) we can deduce

$$\lambda x.student(x) \in s_1 : T \qquad (25)$$

We now show $s_1 : R$. First by (4) we have (6). Applying the β- reduction rule (10) to (6) and (25) we have $s_1 = student(Taroo) : R$. Therefore, the discourse (19) is correct. We now consider the following discourse consisted of a single sentence.

$$Taroo\ ga\ gakusei\ des - u. \qquad (26)$$

In the above sentence type checking fails as follows. Since the postposition *ga* is atatched to *Taroo*, we have (6). Therefore, $\lambda x.student(x) \in s_0$ must have a type of either T or R. However this is impossible. Since $gakusei\ des - u$ can not be used in a sentence of neutral description, $\lambda x.student(x) \in s_i$ never has a type R. The sentence "$x\ ga\ gakusei\ des - u$" always means that "*It is Taroo who is a student*" and is used only in the situation where *gakusei* is a theme. According to Kuno, this use of predicate is called the exhaustive-listing. On the other hand, $\lambda x.student(x)$ can have a type T only when *student* has appeared as in (21) in the preceeding context and the postulate (24) can be used. Since (26) does not have a preceeding text, it never happens. Thus it fails to prove $s_0 : R$ and it has been established that (26) is not a correct discourse.

We now briefly describe an application of our type theoretic approach to the pragmatic problem considered in the previous section. Let us consider the discourse given by (1.24). We rewrite it here in Japanese.

$$Jitensha\ ga\ arimas-u. \tag{27.1}$$

$$Sadoru\ wa\ atarashii\ des-u. \tag{27.2}$$

(27.1) has the same structure as (19.1) and thus its corresponding logical form given by

$$s_0 = (\exists x) bycicle(x) \wedge here\_is(x) \tag{28}$$

has a type R. Next, (27.2) is represented as follows:

$$(\exists! x) saddle(x) \wedge new(x) \tag{29}$$

Here $\exists! x$ means that there exists only one x. Let x_0 be a constant that satisfies (29). Then, corresponding to (27.2), we must prove

$$s_1 = new(x_0) : R \tag{30}$$

Since new $(atarashii\ des - u)$ allows

$$\lambda x.new(x) \in s_1 : T \rightarrow R \tag{31}$$

we can resolve the correctness of the discourse if

$$x_0 \in s_1 : T \tag{32}$$

To show this we introduce the following two postulates:

$$Qxf(x) \in s_0 : R, f(y) \vdash y \in s_0 : R \tag{33}$$

where Q is a quantifier.

$$(\exists y) part\_of(x, y), y \in s_0 : R \vdash x \in s_1 : T \tag{34}$$

Let y_0 be a constant that satisfies (28), then from (23), (28) and (33), we have

$$bycicle(y_0), y_0 \in s_0 : R \tag{35}$$

Thus (32) can be reduced and the proof of the correctness of the discourse will be completed if x_0 satifies $part\_of(x_0, y_0)$. However this is true if we have the postulate (27.1) of Section 2.

So far we have considered discourses consisted of two sentences. However the above method can be easily extended to a discourse that is consisted of more than three sentences. In this case, the inference rules used across several sentences are modified. For example, (16) can be modified as follows:

$$\lambda f.f(Taroo) \in s_i, i < j : (T, R) \rightarrow R$$
$$\vdash Taroo \in s_j : T \tag{16'}$$

where s_i denotes the logical form corresponding to the i-th sentence of a discourse. Furthermore, "$Taroo$" can be arbitrary term, and thus we can establish the following more general rule:

$$\lambda f.f(t) \in s_i, i < j : (T, R) \rightarrow R \vdash t \in s_j : T \tag{16''}$$

4 Conclusions

In this paper, we have given two formal representations of the T-R structure of a natural language discourse. We have proposed using two different notions. The tool used in Section 2 was a logical notation called ontological promiscuity, which is first-order and nonintensional. Using this description, a proof process of utterance interpretation of a discourse has been obtained. It has been shown that the *closed world assumption* (CWA) is essentially related to the proposed proof process. In Section 3, we consider the same problem mainly for Japanese, using another tool. The tool proposed has been typed λ-calculus. A logical notation has been seen as a typed λ-term. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. It is interesting that two concepts similar to those used in this paper are used in the theory of constructive mathematics (**r**-realizability and constructive type theory).

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